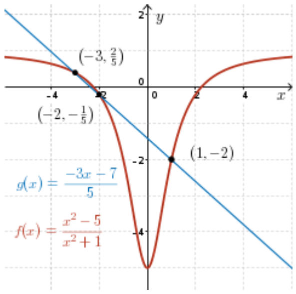
Solving Rational Inequalities

Exercises

1. a. Given the graphs of $f(x) = \dfrac{x^2 - 5}{x^2 + 1}$ and $g(x) = \dfrac{-3x - 7}{5}$, determine the solution of $\dfrac{x^2 - 5}{x^2 + 1} < \dfrac{-3x - 7}{5}$.

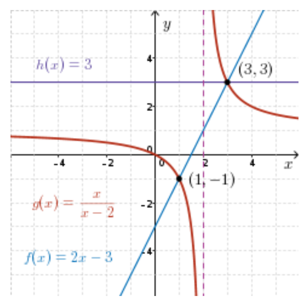


b. Given the graphs of $f(x) = 2x - 3$, $g(x) = \dfrac{x}{x - 2}$ and $h(x) = 3$, determine the solution of

i. $\dfrac{x}{x - 2} \geq 2x - 3$

ii. $\dfrac{x}{x - 2} \leq 3$

iii. $2x - 3 \leq \dfrac{x}{x - 2} \leq 3$



2. Solve each inequality graphically, where $x \in \mathbb{R}$.

a. $\dfrac{3}{x + 2} \geq x$

b. $\dfrac{x}{x + 7} < \dfrac{-x}{x - 2}$

3. Solve each of the following rational inequalities using graphing technology, where $x \in \mathbb{R}$.

a. $\dfrac{10x + 6}{4x + 3} < \dfrac{7x - 4}{3x - 2}$

b. $\dfrac{x}{2x - 8} \geq \dfrac{x^2 + x - 6}{x + 2}$

4. Solve each inequality algebraically. State the solution using interval notation, where $x \in \mathbb{R}$.

a. $\dfrac{3x + 4}{2x - 1} > 0$

b. $\dfrac{12x^2 + 11x + 2}{2x^2 - 7x + 3} \leq 0$

c. $\dfrac{3 - x}{2x + 2} > \dfrac{x}{2}$

d. $\dfrac{1}{-x^2 - 1} < -\dfrac{1}{4}$

e. $\dfrac{2x}{x - 1} \leq \dfrac{x}{x + 2}$

f. $\dfrac{3}{x - 2} - \dfrac{x - 3}{x + 1} > \dfrac{x}{x - 2}$

g. $\dfrac{x}{x^2 - 4} \geq \dfrac{-1}{x + 1}$

5. Cocoa Cascade has determined that the cost of producing their hot chocolate powder can be modelled by $C(x) = \dfrac{60x}{x^2 + 3x + 2}$, where $C(x)$ is

the total cost, in hundreds of thousands of dollars, and x is the number of kilograms of hot chocolate powder, in thousands. The revenue generated from selling the hot chocolate can be modelled by $R(x) = \dfrac{90x}{x^2 + 6x + 8}$, where $R(x)$ is the total revenue in hundreds of thousands of dollars. How many kilograms of hot chocolate powder must Cocoa Cascade sell in order to make a profit?

6. A closed-topped cylindrical tin can is to be made with a volume of 25 cm^3.
   a. Determine a function to model the height, h, of the can in terms of the radius, r.
   b. Determine a function to model the surface area, A, of the tin can in terms of the radius r.
   c. What is an appropriate domain for the surface area function, $A(r)$?
   d. Using graphing technology, determine the values of the radius that will produce a surface area no more than the area of a letter sized piece of paper measuring $21.6 \text{ cm} \times 27.9 \text{ cm}$.
7. The concentration, in milligrams per millilitre, of a vaccination in a patient's bloodstream can be represented by $C(t) = \dfrac{0.36t}{t^2 + 3}$, where $C(t)$ is the concentration and t is the time, in hours, since the vaccination was injected. For how many hours will the concentration of the vaccination be greater than or equal to 0.05 mg/mL?
8. Given $f(x) = \dfrac{2x}{1 - x}$, determine the values of x for which $f(f(x)) \leq -\frac{3}{2}, x \in \mathbb{R}$.
9. For which positive integer values of n is $\dfrac{1}{n+2} \leq 0.02014 \leq \dfrac{1}{n}$ true?
10. Solve $\left|\dfrac{x+4}{x-3}\right| \leq 3$, where $x \in \mathbb{R}$.

Solving Rational Inequalities

Partial Solutions

1. There is no solution provided for this question.

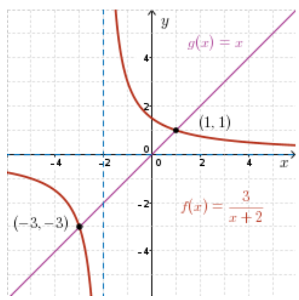
2. a. Let $f(x) = \dfrac{3}{x+2}$ and $g(x) = x$. Graph $f(x)$ and $g(x)$, and determine the point(s) of intersection of the two functions.

To obtain the graph of $f(x) = \dfrac{3}{x+2}$ apply a vertical stretch, from the x-axis, by a factor of 3 and a horizontal translation 2 units left to the graph of $y = \dfrac{1}{x}$. Note that $f(x) = \dfrac{3}{x+2}$ has a vertical asymptote of $x = -2$ and a horizontal asymptote of $y = 0$.

The point(s) of intersection occur when $f(x) = g(x)$.

$$\begin{aligned} \frac{3}{x+2} &= x, x \neq -2 \\ 3 &= x(x+2) \\ 0 &= x^2 + 2x - 3 \\ 0 &= (x+3)(x-1) \\ x &= -3, 1 \\ g(-3) &= -3 \\ g(1) &= 1 \end{aligned}$$

Therefore, the points of intersection are $(-3, -3)$ and $(1, 1)$, giving



Using the graph above, we can solve $\dfrac{3}{x+2} \geq x$. That is, we can determine the intervals on which $f(x) \geq g(x)$. The solution is $x \leq -3$ or $-2 < x \leq 1, x \in \mathbb{R}$.

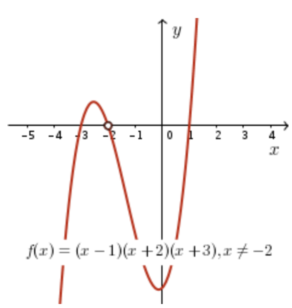
Alternate Approach

Manipulating the inequality first,

$$\begin{aligned} \frac{3}{x+2} &\geq x, x \neq -2 \\ \frac{3}{x+2} - x &\geq 0 \\ \frac{3 - x(x+2)}{x+2} &\geq 0 \\ \frac{-x^2 - 2x + 3}{x+2} &\geq 0 \\ \frac{(x-1)(x+3)}{x+2} &\leq 0 \\ \left[\frac{(x-1)(x+3)}{x+2}\right](x+2)^2 &\leq 0(x+2)^2 \\ (x-1)(x+3)(x+2) &\leq 0, x \neq -2 \end{aligned}$$

To solve this inequality, sketch the graph of $f(x) = (x-1)(x+3)(x+2), x \neq -2$ and determine when $f(x) \leq 0$.

The zeros of this cubic polynomial function are $x = -3, -2$ and 1 (all of multiplicity one), however, $x \neq -2$ so a hole occurs at $(-2, 0)$. The leading coefficient is positive (highest degree term is x^3). This odd degree polynomial function will have opposite end behaviours with $y \to -\infty$ as $x \to -\infty$ and $y \to \infty$ as $x \to \infty$. Sketching the graph of the function using this information, we obtain



Now, $\dfrac{3}{x+2} \geq x$ when $f(x) \leq 0$. Therefore the solution is $x \leq -3$ or $-2 < x \leq 1, x \in \mathbb{R}$.

b. Let $f(x) = \dfrac{x}{x+7}$ and $g(x) = \dfrac{-x}{x-2}$. Graph $f(x)$ and $g(x)$, and determine the point(s) of intersection.

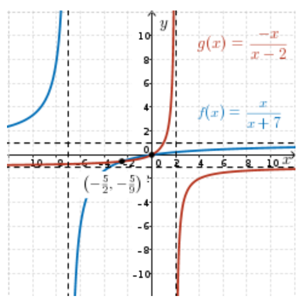
Graphing $f(x) = \dfrac{x}{x+7}, x \neq -7$: Since $f(-7) = \dfrac{-7}{0}$, an undefined value, a vertical asymptote occurs at $x = -7$. As $x \to \pm\infty, f(x) \to \dfrac{x}{x}$ so $f(x) \to 1$ and the horizontal asymptote is $y = 1$. Since $f(x) = 0$ when $x = 0$ the function crosses the x- and y-axis at $(0, 0)$.

Graphing $g(x) = \dfrac{-x}{x-2}, x \neq 2$: Since $g(2) = \dfrac{-2}{0}$, an undefined value, a vertical asymptote occurs at $x = 2$. As $x \to \pm\infty, g(x) \to \dfrac{-x}{x}$ so $g(x) \to -1$ and the horizontal asymptote is $y = -1$. Since $g(x) = 0$ when $x = 0$ the function crosses the x- and y-axis at $(0, 0)$.

The points of intersection occur when $f(x) = g(x)$.

$$\begin{aligned}
\frac{x}{x+7} &= \frac{-x}{x-2}, x \neq -7, 2 \\
(x+7)(x-2)\left(\frac{x}{x+7}\right) &= (x+7)(x-2)\left(\frac{-x}{x-2}\right) \\
x(x-2) &= -x(x+7) \\
x^2 - 2x &= -x^2 - 7x \\
2x^2 + 5x &= 0 \\
x(2x+5) &= 0 \\
x &= 0, -\frac{5}{2} \\
f(0) &= 0 \\
f\left(-\frac{5}{2}\right) &= -\frac{5}{9}
\end{aligned}$$

Therefore, the points of intersection are $(0, 0)$ and $\left(-\frac{5}{2}, -\frac{5}{9}\right)$. Sketching the graph of the function using this information, we obtain



Using the graph, we can solve $\dfrac{x}{x+7} < \dfrac{-x}{x-2}$. That is, we can determine the intervals on which $f(x) < g(x)$. The solution is $-7 < x < -\frac{5}{2}$ or $0 < x < 2, x \in \mathbb{R}$.

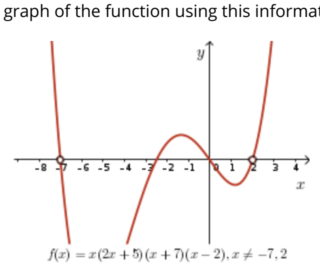
Alternate Approach

Manipulating the inequality first,

$$\begin{aligned}
\frac{x}{x+7} &< \frac{-x}{x-2}, x \neq -7, 2 \\
\frac{x}{x+7} + \frac{x}{x-2} &< 0 \\
\frac{x(x-2) + x(x+7)}{(x+7)(x-2)} &< 0 \\
\frac{2x^2 + 5x}{(x+7)(x-2)} &< 0 \\
\frac{2x^2 + 5x}{(x+7)(x-2)}(x+7)^2(x-2)^2 &< 0(x+7)^2(x-2)^2 \\
x(2x+5)(x+7)(x-2) &< 0, x \neq -7, 2
\end{aligned}$$

To solve this inequality, sketch the graph of $f(x) = x(2x+5)(x+7)(x-2), x \neq -7, 2$ and determine when $f(x) < 0$.

The zeros of this quartic polynomial function are $-7, -\dfrac{5}{2}, 0$ and 2 (all of multiplicity one), however, $x \neq -7, 2$ so holes occur at $(-7, 0)$ and $(2, 0)$. The leading coefficient is positive (highest degree term is $2x^4$). This even degree polynomial function will have same end behaviours with $y \to \infty$ as $x \to \pm\infty$. Sketch the graph of the function using this information.



Now, $\dfrac{x}{x+7} < \dfrac{-x}{x-2}$ when $f(x) < 0$. Therefore, the solution is $-7 < x < -2.5$ or $0 < x < 2, x \in \mathbb{R}$.

3. There is no solution provided for this question.

4. a. The inequality, $\dfrac{3x+4}{2x-1} > 0, x \neq \frac{1}{2}$, is already arranged in simplified form with all of the terms on one side of the inequality. The zero values of the factors in the numerator and denominator are $-\frac{4}{3}$ and $\frac{1}{2}$. This gives the following interval sign table:

	$x < -\frac{4}{3}$	$-\frac{4}{3} < x < \frac{1}{2}$	$x > \frac{1}{2}$
$3x+4$	$-$	$+$	$+$
$2x-1$	$-$	$-$	$+$
$\dfrac{3x+4}{2x-1}$	$+$	$-$	$+$

We see that $\dfrac{3x+4}{2x-1} > 0$ for $x \in (-\infty, -\frac{4}{3}) \cup (\frac{1}{2}, \infty), x \in \mathbb{R}$.

b. The inequality, $\dfrac{12x^2+11x+2}{2x^2-7x+3} \leq 0$, is already arranged with all of the terms on one side of the inequality. Factoring both numerator and denominator, $f(x) = \dfrac{12x^2+11x+2}{2x^2-7x+3} = \dfrac{(3x+2)(4x+1)}{(2x-1)(x-3)}, x \neq \frac{1}{2}, 3$.

The zero values of the factors in the numerator and denominator are $-\frac{2}{3}, -\frac{1}{4}, \frac{1}{2}$ and 3. This gives the following interval table:

	$x < -\frac{2}{3}$	$-\frac{2}{3} < x < -\frac{1}{4}$	$-\frac{1}{4} < x < \frac{1}{2}$	$\frac{1}{2} < x < 3$	$x > 3$
$3x+2$	$-$	$+$	$+$	$+$	$+$
$4x+1$	$-$	$-$	+	+	+
2x - 1	-	-	-	+	+
x - 3	-	-	-	-	+
\dfrac{(3x + 2)(4x + 1)}{(2x - 1)(x - 3)}	+	-	+	-	+

We see that \dfrac{12x^2+11x+2}{2x^2-7x+3}\leq0 for x\in \left[-\frac{2}{3},-\frac{1}{4}\right] \cup \left(\frac{1}{2},3 \right), x \in\mathbb{R} .

c. Collect all terms to one side of the inequality. Simplify to a single expression in factored form, as shown below. \begin{align*} \dfrac{3-x}{2x+2}& \gt \dfrac{x}{2}\\ \dfrac{3-x}{2x+2}-\dfrac{x}{2} & \gt 0\\ \dfrac{(3-x)-x(x+1)}{2(x+1)} & \gt 0\\ \dfrac{3-x-x^2-x}{2(x+1)} & \gt 0\\ \dfrac{-x^2-2x+3}{2(x+1)} & \gt 0\\ \dfrac{(x+3)(x-1)}{2(x+1)} & \lt 0, x\neq -1\\ \end{align*}

The zero values of the factors in the numerator and denominator are -3, -1 , and 1 . This gives the following interval table:

	x \lt -3	-3 \lt x \lt -1	-1 \lt x \lt 1	x \gt 1
x + 3	-	+	+	+
x - 1	-	-	-	+
2(x + 1)	-	-	+	+
\dfrac{(x + 3)(x - 1)}{2(x + 1)}	-	+	-	+

From the interval table above, \dfrac{3-x}{2x+2} \gt \dfrac{x}{2} , or \dfrac{(x+3)(x-1)}{2(x+1)} \lt 0 , for x\in(-\infty,-3)\cup(-1,1), x \in\mathbb{R} .

d. Collect all terms to one side of the inequality. Simplify to a single expression in factored form, as shown below. \begin{align*} \dfrac{1}{-x^2-1}&\lt -\dfrac{1}{4}\\ \dfrac{1}{-x^2-1}+\dfrac{1}{4} &\lt 0\\ \dfrac{4+(-x^2-1)}{-4(x^2+1)} &\lt 0\\ \dfrac{-(x^2-3)}{-4(x^2+1)} &\lt 0\\ \dfrac{x^2-3}{4(x^2+1)} &\lt 0 \end{align*} The zero values of the factors in the numerator and denominator are \sqrt{3} and -\sqrt{3} (x^2-3=0 when x=\pm\sqrt{3} and x^2+1\neq0). This gives the following interval table:

	x \lt -\sqrt{3}	-\sqrt{3} \lt x \lt \sqrt{3}	x \gt \sqrt{3}
x^2 - 3	+	-	+
4(x^2 + 1)	+	+	+

$\dfrac{x^2 - 3}{4(x^2 + 1)}$	+	-	+

From the interval table above we see $\dfrac{1}{-x^2-1} \lt -\dfrac{1}{4}$, that is $\dfrac{x^2-3}{4(x^2+1)} \lt 0$, for $x\in(-\sqrt{3},\sqrt{3}), x \in\mathbb{R}$.

e. Collect all terms to one side of the inequality. Simplify to a single expression in factored form, as shown below. $$\begin{align*} \dfrac{2x}{x-1}&\leq \dfrac{x}{x+2}\\ \dfrac{2x}{x-1}-\dfrac{x}{x+2} &\leq 0\\ \dfrac{2x(x+2)-x(x-1)}{(x+2)(x-1)} &\leq 0\\ \dfrac{x^2+5x}{(x+2)(x-1)} &\leq 0\\ \dfrac{x(x+5)}{(x+2)(x-1)} &\leq 0, x \neq -2, 1 \end{align*}$$ The zero values of the factors in the numerator and denominator are -5 , -2 , 0 and 1 . This gives the following interval table:

	$x \lt -5$	$-5 \lt x \lt -2$	$-2 \lt x \lt 0$	$0 \lt x \lt 1$	$x \gt 1$
x	-	-	-	+	+
$x + 5$	-	+	+	+	+
$x + 2$	-	-	+	+	+
$x - 1$	-	-	-	-	+
$\dfrac{x(x + 5)}{(x + 2)(x - 1)}$	+	-	+	-	+

We see that $\dfrac{2x}{x-1}\leq\dfrac{x}{x+2}$, that is $\dfrac{x(x+5)}{(x+2)(x-1)}\leq0$, for $x\in[-5,-2)\cup[0,1), x \in\mathbb{R}$.

f. Collect all terms to one side of the inequality. Simplify to a single expression in factored form, as shown below. $$\begin{align*} \dfrac{3}{x-2}-\dfrac{x-3}{x+1}&\gt \dfrac{x}{x-2}\\ \dfrac{3}{x-2}-\dfrac{x-3}{x+1}-\dfrac{x}{x-2} &\gt 0\\ \dfrac{3-x}{x-2}-\dfrac{x-3}{x+1} &\gt 0\\ \dfrac{(3-x)(x+1)-(x-3)(x-2)}{(x-2)(x+1)} &\gt 0\\ \dfrac{-(2x^2-7x+3)}{(x-2)(x+1)} &\gt 0\\ \dfrac{(2x-1)(x-3)}{(x-2)(x+1)} &\lt 0, x\neq -1, 2\\ \end{align*}$$

The zero values of the factors in the numerator and denominator are -1 , $\frac{1}{2}$, 2 and 3 . This gives the following interval table:

	$x \lt -1$	$-1 \lt x \lt \frac{1}{2}$	$\frac{1}{2} \lt x \lt 2$	$2 \lt x \lt 3$	$x \gt 3$
$2x - 1$	-	-	+	+	+
$x - 3$	-	-	-	-	+
$x - 2$	-	-	-	+	+
$x + 1$	-	+	+	+	+
$\dfrac{(2x - 1)(x - 3)}{(x - 2)(x + 1)}$	+	-	+	-	+

We see that $\dfrac{3}{x-2}-\dfrac{x-3}{x+1} \gt \dfrac{x}{x-2}$, that is $\dfrac{(2x-1)(x-3)}{(x-2)(x+1)} \lt 0$, for $x\in \left(-1,\frac{1}{2}\right) \cup (2,3), x \in\mathbb{R}$.

g. Collect all terms to one side of the inequality. Simplify to a single expression in factored form, as shown below. $$\begin{align*} \dfrac{x}{x^2-4}&\geq \dfrac{-1}{x+1}\\ \dfrac{x}{x^2-4}+ \dfrac{1}{x+1}&\geq 0\\ \dfrac{x(x+1)+(x^2-4)}{(x+1)(x+2)(x-2)} &\geq 0\\ \dfrac{2x^2+x-4}{(x+1)(x+2)(x-2)} &\geq 0, x\neq -2,-1,2 \end{align*}$$ Use the quadratic formula to determine when $2x^2+x-4 =0$. The zero values for the numerator and denominator are -2 , $\frac{-1-\sqrt{33}}{4}$, -1 , $\frac{-1+\sqrt{33}}{4}$ and 2 . This gives the following interval table:

	$x \lt -2$	$-2 \lt x \lt \tfrac{-1 - \sqrt{33}}{4}$	$\tfrac{-1 - \sqrt{33}}{4} \lt x \lt -1$	$-1 \lt x \lt \tfrac{-1 + \sqrt{33}}{4}$	$\tfrac{-1 + \sqrt{33}}{4} \lt x \lt 2$	$x \gt 2$
$2x^2 + x - 4$	+	+	-	-	+	+
$x + 1$	-	-	-	+	+	+
$x + 2$	-	+	+	+	+	+
$x - 2$	-	-	-	-	-	+
$\dfrac{2x^2 + x - 4}{(x + 1)(x + 2)(x - 2)}$	-	+	-	+	-	+

We see that $\dfrac{x}{x^2-4}\geq \dfrac{-1}{x+1}$, that is $\dfrac{2x^2+x-4}{(x+1)(x+2)(x-2)}\geq0$, for $x \in \left(-2,\frac{-1-\sqrt{33}}{4} \right] \cup \left(-1, \frac{-1 + \sqrt{33}}{4} \right] \cup \left(2, \infty \right), x\in\mathbb{R}$.

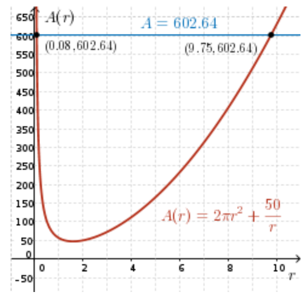
5. There is no solution provided for this question.

6. a. The volume of a cylinder is given by the formula: $V=\pi r^2 h$. So, $h(r) = \dfrac{V}{\pi r^2}=\dfrac{25}{\pi r^2}$.

   b. The surface area of a cylinder is given by the formula $A(r)=2 \pi r h + 2 \pi r^2 = 2 \pi r \left(\dfrac{25}{\pi r^2}\right)+2\pi r^2$. Therefore $A(r)= \dfrac{50}{r} + 2\pi r^2$.

c. Since the radius cannot be a negative value and $r \neq 0$ for the equation $A(r)$, an appropriate domain for the function $A(r)$ is: $\{r\in \mathbb{R} \mid r \gt 0\}$.

d. A letter-sized piece of paper measuring $21.6~\text{cm}$ by $27.9~\text{cm}$ has a surface area of $602.64~\text{cm}^2$. So, we want to determine r such that $\dfrac{50}{r} + 2\pi r^2 \lt 602.64$. The functions $A(r)=\dfrac{50}{r} + 2\pi r^2$ and $A=602.64$ are graphed below.



From this we can see that the surface area of a tin can will be no more than the area of a letter-sized piece of paper when the radius is between approximately $0.08~\text{cm}$ and $9.75~\text{cm}$ $\left(r \in (0.08, 9.75), r\in\mathbb{R} \right)$.

7. There is no solution provided for this question.

8. First, determine $f(f(x))$ and identify any restrictions on x . $\begin{align*} f(f(x)) &= f\left(\dfrac{2x}{1-x}\right)\\ &= \dfrac{2\left(\dfrac{2x}{1-x}\right)}{1-\left(\dfrac{2x}{1-x}\right)} \hspace{2cm} x \neq 1 \text{ and } 1-\left(\dfrac{2x}{1-x}\right)\neq0\\ &= \dfrac{\dfrac{4x}{1-x}}{\dfrac{(1-x)-2x}{1-x}}\\ &= \dfrac{\dfrac{4x}{1-x}}{\dfrac{1-3x}{1-x}}\\ &= \dfrac{4x}{1-x} \cdot \dfrac{1-x}{1-3x} \hspace{2cm} x \neq \dfrac{1}{3}, 1\\ &= \dfrac{4x}{1-3x}, x\neq\dfrac{1}{3}, 1 \end{align*}$ To solve $\dfrac{4x}{1-3x}\leq -\dfrac{3}{2}$, collect all terms to one side of the inequality and simplify to a single expression in factored form. $\begin{align*} \dfrac{4x}{1-3x} &\leq -\dfrac{3}{2}\\ \dfrac{4x}{1-3x}+ \dfrac{3}{2} &\leq 0\\ \dfrac{2(4x)+3(1-3x)}{2(1-3x)}&\leq 0\\ \dfrac{3-x}{2(1-3x)} &\leq 0 \end{align*}$ The zero values of the factors in the numerator and denominator are $\frac{1}{3}$ and 3 . This gives the following interval table:

	$x \lt \frac{1}{3}$	$\frac{1}{3} \lt x \lt 3$	$x \gt 3$
3 - x	+	+	-
2(1 - 3x)	+	-	-
$\dfrac{3 - x}{2(1 - 3x)}$	+	-	+

We must also take into consideration the restriction, $x\neq 1$, from the original function, $f(x) = \dfrac{2x}{1-x}$. So, $f(f(x))\leq-\frac{3}{2}$, $x\in\mathbb{R}$ when $\frac{1}{3} \lt x \lt 1$ or $1 \lt x \leq 3, x\in\mathbb{R}$.

9. There is no solution provided for this question.

10.

$$\begin{align*} \left\lvert \dfrac{x+4}{x-3} \right\rvert &\leq 3\\ -3 \leq \dfrac{x+4}{x-3} &\leq 3, x \neq 3 \end{align*}$$

We need to determine when $\dfrac{x+4}{x-3}\geq -3$ and when $\dfrac{x+4}{x-3}\leq 3$. For $\dfrac{x+4}{x-3}\geq -3$, we have $\begin{align*} \dfrac{x+4}{x-3}&\geq -3\\ \dfrac{x+4}{x-3}+3 &\geq 0\\ \dfrac{4x-5}{x-3} &\geq 0, x \neq 3 \end{align*}$

The zero values of the factors in the numerator and denominator are $\frac{5}{4}$ and 3. This gives the following interval table:

	$x \lt \frac{5}{4}$	$\frac{5}{4} \lt x \lt 3$	$x \gt 3$
4x - 5	-	+	+
x - 3	-	-	+
$\dfrac{4x - 5}{x - 3}$	+	-	+

So, $\dfrac{x+4}{x-3}\geq -3$, that is $\dfrac{4x-5}{x-3}\geq0$, for $x\leq\dfrac{5}{4}$ or $x\gt 3, x\in\mathbb{R}$.

For $\dfrac{x+4}{x-3}\leq 3$, we have $\begin{align*} \dfrac{x+4}{x-3}&\leq 3\\ \dfrac{x+4}{x-3}-3 &\leq 0\\ \dfrac{13-2x}{x-3} & \leq 0, x \neq 3 \end{align*}$

The zero values of the factors in the numerator and denominator are 3 and $\frac{13}{2}$. This gives the following interval table:

	$x \lt 3$	$3 \lt x \lt \frac{13}{2}$	$x \gt \frac{13}{2}$
13 - 2x	+	+	-
x - 3	-	+	+
$\dfrac{13 - 2x}{x - 3}$	-	+	-

So, $\dfrac{x+4}{x-3}\leq 3$, that is $\dfrac{13-2x}{x-3}\leq0$, for $x\lt 3$ or $x\geq \dfrac{13}{2}, x\in\mathbb{R}$.

The solution to the inequality $\left\lvert \dfrac{x+4}{x-3} \right\rvert \leq 3$ must satisfy both solutions, ($x \leq \frac{5}{4}$ or $x\gt 3$) AND ($x\lt 3$ or $x\geq \dfrac{13}{2}$).

Therefore, $\left\lvert \dfrac{x+4}{x-3} \right\rvert \leq 3$ for $x\leq\frac{5}{4}$ or $x\geq\frac{13}{2}$, $x\in\mathbb{R}$.